

Perturbative superstrata

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Abstract

We study a particular class of D-brane bound states in type IIB string theory (dubbed “superstrata”) that describe microstates of the 5D Strominger–Vafa black hole. By using the microscopic description in terms of open strings we probe these configurations with generic light closed string states and from there we obtain a linearized solution of six-dimensional supergravity preserving four supersymmetries. We then discuss two generalizations of the solution obtained which capture different types of non-linear corrections. By using this construction, we can provide the first explicit example of a superstratum solution which includes the effects of the KK-monopole dipole charge to first order.

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1. Introduction

In string theory supersymmetric black holes are realized as bound states at threshold of many basic constituents, such as perturbative string states and branes. In [1,2] this picture was used to count in concrete examples the degeneracy of the configurations that have the same (three) conserved charges. In particular, the setup studied in those papers is type IIB string theory compactified on an S^1 of radius $R \gg \sqrt{\alpha'}$ times a string-sized four manifold, which is either T^4 or K_3 . In the large charge limit the microscopic counting matches perfectly the Bekenstein–Hawking entropy of a black hole solution with the same charges. However, while the study of

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the bound state degeneracy is performed at zero string coupling g_s , the gravitational backreaction of the string/brane bound state, and thus its connection to black holes, is manifest only when $g_s N$ is big, where N roughly indicates the number of elementary constituents of each type present in the bound state. The presence of four preserved supercharges is crucial for connecting the degeneracy of the configuration at $g_s = 0$ with the black hole entropy derived from the black hole geometry. In [3–5] a new line of research was initiated with the aim to understand the gravitational backreaction of the different configurations (known as microstates), which at the level of the free theory account for the bound state degeneracy. One of the aims of this programme is to understand whether it is possible to give a supergravity description of each microstate in the limit when g_s is small, but $g_s N$ is finite. Even if this geometric description fails when approaching the bound state under analysis, the really important question is whether this happens before a horizon is formed or not.

Even though there has been continuous progress over the last ten years and many works have been published on the subject (see for instance the review articles [6–12]), a complete understanding of the 3-charge microstate geometries relevant for the black hole studied in [1,2] is still lacking. The question of whether the generic microstate can be described by a horizonless geometry is still open. However very far away from the location of the bound state, the geometry will be certainly described by a 5D Minkowski metric times the compact space plus tiny corrections. There are cases where this asymptotically flat part can be glued with a smooth geometry describing the gravitational backreaction of a particular 3-charge bound state at any value of the radial distance r , see [13–15] for some of the first examples of such geometries. These configurations appear as 1/8-BPS solutions of the standard type IIB supergravity equations, but with some striking geometric properties. In this case one can check explicitly that no horizons develop inside the throat of the geometry and the sources appear to have dissolved into fluxes of the supergravity gauge fields, which makes the classical solution regular even in the interior. A more general class of 1/8-BPS solutions with the same features was constructed in [16,17]. The set of 3-charge microstates with a known gravitational dual was further extended in [18]: the main novelty of this new class of backgrounds is that the large S^1 present in the compact space plays a non-trivial role and the solutions are genuinely six-dimensional.

In this paper we will follow the approach of [19–21] and use the microscopic description of the bound state in terms of D-branes and open string states to derive the geometric properties of 3-charge microstates in particular limits. The basic idea of this approach is that the couplings of the bound state with the massless closed string sector of the theory (i.e. the supergravity fields) are described in the underlying World-sheet Conformal Field Theory (WCFT) by a set of correlation functions with disk topology. The conditions imposed on the boundary of the disk and the possible presence of open string states define the particular microstate under analysis. The WCFT correlators we are interested in will always have a single external closed string state. These correlators are directly related to the backreaction of the D-brane bound state for the supergravity field corresponding to the closed string state considered. Physically this closed string is the probe used to explore the backreacted geometry. In [19–21] this probe was always taken to have zero momentum in all compact directions. Of course, the information derived in this way cannot distinguish between localized or smeared configurations and the supergravity solutions derived could be interpreted entirely in a (non-minimal) five-dimensional supergravity [22].

We wish to revisit this analysis by allowing closed string probes that are localized in the large S^1 of the compact space, or in other words that have momentum along this S^1 . In particular, we will still focus on the D-brane configurations discussed in [21], but we wish to explore the geometry in a finer detail by using the more general probes mentioned above (of course the

same generalization can be done for other D-brane configurations). Our analysis shows that, in the D1–D5 frame, the 3-charge microstates behave differently from the 2-charge ones: while in this second case the use of localized probes does not give any new information, the 3-charge microstates seem to always have a non-trivial dependence on the S^1 coordinate. A purely 5D geometry is obtained only if we focus just on delocalized probes along the S^1 or, in other words, we smear over the S^1 . In principle, this process of smearing can induce spurious singularities which would be absent in the complete geometry and it is an interesting open problem to understand exactly when this happens. The smeared configurations are likely to be described by exotic or non-geometric configurations, of the type studied in [23,24].

The bound states we are interested in are constructed at the microscopic level by taking D5 branes wrapped on the whole compact space and D1 branes wrapped on the S^1 and by giving them an identical profile describing a vibration in the transverse non-compact space. As usual, in order to preserve some supercharges, the functions f^i describing the shape of the D-branes in the transverse directions can depend only on either one of the two light-cone coordinates, v and u , constructed out of time and the S^1 coordinate, but not on both. In order to have a real bound state, one should switch on a non-trivial KK-monopole dipole charge which at the microscopic level corresponds to give a non zero vacuum expectation value (vev) to some D1/D5 open string states [19]. It was argued in [25], mostly based on supersymmetry arguments, that this class of bound states should be described by smooth supergravity configurations parameterized by arbitrary functions of two variables, that were dubbed “superstrata”. The construction of the exact supergravity solutions for superstrata is an important open problem: the first steps in this direction were taken in [26,27] (building on previous supergravity results of [28,29]), which derived exact supergravity solutions representing a superposition of D1 and D5 branes with generic oscillation profiles but no KK-monopole dipole charge. In both the WCFT and supergravity approaches it is easier to start by treating the KK-monopole dipole charge perturbatively. The main goal of this paper is to provide the first explicit example of a solution which includes the effects of the KK-monopole dipole charge to first order. It is interesting to notice that the supergravity solutions emerging from the simplest D-brane configurations studied here by following [21] do not fall in the ansatz considered in [26]. It should be possible to engineer a D-brane configuration whose backreaction contains only the fields of the restricted ansatz [26], but this will involve a more complicated choice of the open string vev’s defining microscopically the bound state. Thus simpler microscopic configurations correspond to more complicated supergravity solutions and vice versa. This might be somewhat unexpected or be just a result of the fine tuning required at the microscopic level to cancel the extra dipoles which are allowed by supersymmetry but are absent in the ansatz [26].

The paper is structured as follows. In Section 2 we generalize the ansatz discussed in [22] so as to adapt it to the v -dependent case we are interested in. The full list of constraints imposed by supersymmetry and the equations of motion on the functions appearing in the ansatz is not known. Work is in progress to derive these equations from first principles [30]. However it is not difficult to start from the equations derived in [26] and generalize them at the linearized level needed for our analysis. In Section 3 we collect the results for the 1-point string correlators mentioned above and extract the geometric information we need by comparing them against the ansatz of Section 2. In Section 3 we provide a first generalization of the results obtained from string theory and show that it is natural to write the linearized supergravity configuration in terms of a set of simple scalar functions and 1-forms. In particular the 1-form β captures the KK-monopole dipole charge of the configuration; in the diagrammatic language of string amplitudes β is related to the disk amplitudes with the insertion of one g_{ui} graviton and an

arbitrary number of twisted open string vertices. We show that the linearized equations in the bulk are satisfied if we assume some simple properties for the basic building blocks of the supergravity configuration. In particular they must enjoy the same harmonic and duality constraints that, in the perturbative string approach of Section 3, follow from the BRST invariance of the open string vertices [19]. In Section 5 we focus on the special case where the functions f^i describing the D-brane profile have periodicity $2\pi R$ (this needs not to be the case when the D-branes are multiply wrapped as it happens for generic 3-charge states). This is the class of microstates studied from a supergravity point of view in [26]. We show that this case is more easily studied in a coordinate system where the metric for the non-compact space is conformally flat, but the 10D metric is not asymptotically Minkowski. In this frame the supergravity equations take a particularly simple form. This allows us to present a further generalization and obtain an explicit solution which includes the non-linear terms in the D1 and D5 charges, but is still linear in the KK-monopole dipole charge. In Section 6 we present our conclusion by discussing some possible further developments and the connections of our approach with recent supergravity literature on the subject.

2. The supergravity ansatz

Eq. (2.8) of [22] contains an explicit ansatz for type IIB supergravity compactified on $S^1 \times T^4$ which preserves 4 supercharges provided that the conditions (2.9)–(2.11) of that paper are satisfied. We now wish to extend that ansatz to the case where all functions and forms appearing in the various fields can depend on v , besides the R^4 coordinates x^i , with

$$v = \frac{t+y}{\sqrt{2}}, \quad u = \frac{t-y}{\sqrt{2}}, \quad (2.1)$$

where t and y indicate the coordinates along the time and the S^1 direction respectively. We will also rephrase the ansatz by using the language of [28,26]. Then the string frame metric takes the following form

$$ds^2 = -2 \frac{\alpha}{\sqrt{Z_1 Z_2}} (dv + \beta) \left(du + \omega + \frac{\mathcal{F}}{2} (dv + \beta) \right) + \sqrt{Z_1 Z_2} ds_4^2 + \sqrt{\frac{Z_1}{Z_2}} ds_{T^4}^2, \quad (2.2)$$

where α , \mathcal{F} and the Z_I 's are functions depending on v and the R^4 coordinates x^i , while ω and β are 1-forms on R^4 but can depend on v as well. The two 4D metrics ds_4^2 and $ds_{T^4}^2$ indicate the non-compact and the T^4 metric respectively. For the time being we allow for a general v -dependent R^4 metric h_{ij} and for the sake of simplicity take the torus to be perfectly cubic

$$ds_4^2 = h_{ij} dx^i dx^j, \quad ds_{T^4}^2 = (dz^1)^2 + \cdots + (dz^4)^2. \quad (2.3)$$

The ansatz for the remaining type IIB supergravity fields is written in terms of a function Z_4 related to α by

$$\alpha = \left(1 - \frac{Z_4^2}{Z_1 Z_2} \right)^{-1}, \quad (2.4)$$

the 1-forms a_1 and a_4 , the 2-forms γ_2 and δ_2 and the 3-form x_3 . All these ingredients, except for x_3 , already appear in the v -independent ansatz [22]. Thus it is useful to summarize the redefinition necessary to map the conventions of that paper with the conventions used here:

$$\beta = \frac{\hat{a}_3}{\sqrt{2}}, \quad \omega = \sqrt{2}\hat{k} - \frac{\hat{a}_3}{\sqrt{2}}, \quad \mathcal{F} = -2(\hat{Z}_3 - 1), \quad a_1 = \sqrt{2}\hat{a}_1, \quad a_4 = \sqrt{2}\hat{a}_4, \quad (2.5)$$

where the hatted quantities are those appearing in [22]. Now we can complete the list of fields appearing in our ansatz. For the dilaton we take

$$e^{2\phi} = \alpha \frac{Z_1}{Z_2}, \quad (2.6)$$

and the NS–NS 2-form is

$$B^{(2)} = -\frac{\alpha Z_4}{Z_1 Z_2} (du + \omega) \wedge (dv + \beta) + a_4 \wedge (dv + \beta) + \delta_2. \quad (2.7)$$

Finally the Ramond–Ramond (RR) forms are

$$\begin{aligned} C^{(0)} &= \frac{Z_4}{Z_1}, \\ C^{(2)} &= -\frac{\alpha}{Z_1} (du + \omega) \wedge (dv + \beta) + a_1 \wedge (dv + \beta) + \gamma_2, \\ C^{(4)} &= \frac{Z_4}{Z_2} dV_{T^4} - \frac{\alpha Z_4}{Z_1 Z_2} \gamma_2 \wedge (du + \omega) \wedge (dv + \beta) + x_3 \wedge (dv + \beta), \end{aligned} \quad (2.8)$$

where $dV_{T^4} = dz^1 \wedge dz^2 \wedge dz^3 \wedge dz^4$.

This generalizes the ansatz studied in [26] by adding the fields $B^{(2)}$, $C^{(0)}$ and $C^{(4)}$, which, in the language of 6D supergravity, should correspond to the addition of an extra tensor multiplet on top of the gravity and tensor multiplet already present in [26]. We leave the detailed analysis of the constraints imposed by supersymmetry and the equations of motion in this more general set up to a forthcoming paper [30]. However, in most of this paper we will be working in the approximation in which only linear terms in the expansion of the geometry around flat space are kept: at this linearized level the new fields present in the ansatz above basically decouple from those already present in the ansatz used in [26]. So we can use the equations discussed in that paper and supplement them with a set of linearized equation for Z_4 , a_4 , δ_2 and x_3 .

Let us denote by d the differential with respect to the R^4 coordinates and define

$$D \equiv d - \beta \wedge \partial_v. \quad (2.9)$$

The first conditions are on the 1-form β and the 4D metric ds_4^2 : β has to satisfy

$$D\beta = \star_4 D\beta, \quad (2.10)$$

where the \star_4 represents the Hodge star according to the R^4 metric h_{ij} ; the Hodge dual with respect to the flat R^4 will instead be denoted as \ast_4 . The metric ds_4^2 has to be “almost hyper-Kähler”, which means that there exist three 2-forms $J^{(A)} \equiv \frac{1}{2} J_{ij}^{(A)} dx^i \wedge dx^j$, with $A = 1, 2, 3$, satisfying

$$J^{(A)} \wedge J^{(B)} = -2\delta^{AB} \star_4 1, \quad dJ^{(A)} = \partial_v (\beta \wedge J^{(A)}). \quad (2.11)$$

This implies that the J ’s are anti-self-dual with respect to the star \star_4 defined above:

$$J^{(A)} = -\star_4 J^{(A)}. \quad (2.12)$$

As usual, by raising one index and choosing an appropriate ordering, we can define three almost complex structures

$$J^{(A)i}{}_k J^{(B)k}{}_j = \epsilon^{ABC} J^{(C)i}{}_j - \delta^{AB} \delta^i{}_j. \quad (2.13)$$

For later use, by using the complex structures we define a new anti-self-dual 2-form ψ

$$\psi \equiv \frac{1}{8} \epsilon^{ABC} J^{(A)ij} \dot{J}^{(B)}_{ij} J^{(C)}, \quad (2.14)$$

where the dot indicates the derivative with the respect to v .

Let us now consider the equations for the part of the ansatz determining the charges and the dipoles of the D1 and D5 branes. Again this sector was already studied in [26] and it turns out that we can use the same¹ set of equations also in our case. In order to put the D1 and the D5 branes on the same footing, let us suppose that the gauge potential $C^{(6)}$ takes a form which closely follows that of $C^{(2)}$ in (2.8)

$$C^{(6)} = \left[-\frac{\alpha}{Z_2} (du + \omega) \wedge (dv + \beta) + a_2 \wedge (dv + \beta) + \gamma_1 \right] \wedge dV_{T^4} + \dots, \quad (2.15)$$

where the dots stand for terms that do not have components along the T^4 . The equations we give below ensure that it is possible to define a 1-form a_2 and a 2-form γ_1 so as to satisfy (2.15). The 2-forms γ_I satisfy

$$D\gamma_2 = \star_4(DZ_2 + \dot{\beta}Z_2) + a_1 \wedge D\beta, \quad D\gamma_1 = \star_4(DZ_1 + \dot{\beta}Z_1) + a_2 \wedge D\beta. \quad (2.16)$$

Then it is convenient to combine a_1 and a_2 in two new 2-forms Θ_1 and Θ_2

$$\Theta_1 = da_1 + \partial_v(\gamma_2 - \beta \wedge a_1), \quad \Theta_2 = da_2 + \partial_v(\gamma_1 - \beta \wedge a_2), \quad (2.17)$$

which must satisfy the following duality conditions involving the 2-form ψ defined in (2.14)

$$\star_4(\Theta_1 - Z_2\psi) = \Theta_1 - Z_2\psi, \quad \star_4(\Theta_2 - Z_1\psi) = \Theta_2 - Z_1\psi. \quad (2.18)$$

The equations for Z_1 and Z_2 are a consequence of (2.16) and (2.17)

$$D \star_4(DZ_2 + \dot{\beta}Z_2) = -\Theta_1 \wedge D\beta, \quad D \star_4(DZ_1 + \dot{\beta}Z_1) = -\Theta_2 \wedge D\beta. \quad (2.19)$$

Now we turn our attention to the equations that are sensitive to the novelty of the ansatz considered in this paper. We will give only the linearized version of these equations. We first have as set of constraints which are the (linearized) analogue of (2.16) and (2.18)

$$d\delta_2 = \star_4 dZ_4, \quad \star_4(da_4 - \dot{\delta}_2) = da_4 - \dot{\delta}_2. \quad (2.20)$$

There is also a constraint for the new component x_3 of the 4-form gauge potential

$$dx_3 = \star_4 \dot{Z}_4. \quad (2.21)$$

Then we have the relation that constrains the angular momentum 1-form ω , that can be derived, for example, by requiring the existence of the RR 6-form $C^{(6)}$. At the linearized level the new fields Z_4, a_4, δ_2, x_3 do not enter this relation, and we can thus read it off from [26]:

$$d\omega + \star_4 d\omega = (\Theta_1 - \psi) + (\Theta_2 - \psi). \quad (2.22)$$

¹ We find it more convenient to work with gauge potentials rather than with field strengths, as instead was done in [26]. Moreover the RR 3-form field strength used here should be identified with twice the 3-form G of [26]: this has the consequence that $2\Theta_I^{\text{there}} = \Theta_I^{\text{here}}$ and also that $2\hat{\psi}^{\text{there}} = \psi^{\text{here}}$. Moreover: $2\gamma_2^{\text{there}} = d(\gamma_2^{\text{here}} + a_1 \wedge \beta)$.

This concludes the conditions following from supersymmetry. They also imply all the second-order equations of motion, except for the vv -component of the Einstein equations. At the linearized order even this extra constraint does not get modified by the new fields, and it reads

$$*_4 d *_4 \left(\dot{\omega} - \frac{1}{2} d\mathcal{F} \right) = \partial_v^2 (Z_1 + Z_2) + \frac{1}{2} \partial_v^2 (h_{ii}). \quad (2.23)$$

3. Mixed disk amplitudes revisited

Let us start from an unbound point-like state whose basic constituents are D1 branes wrapped on the S^1 and D5's wrapped on the whole compact space. All D-branes vibrate in the non-compact space according to the same profile $f_i(v)$. From the WCFT point of view these D-branes can be described by using the boundary state formalism (see [31,32] for a review), as discussed in [33–35]. In [20] this approach was used to show that the boundary state $|B\rangle_{fi}$ contains the information necessary to reconstruct the 2-charge solutions discussed in [36,37] (once they are rewritten in the appropriate duality frame). By following the idea sketched in the Introduction, one can calculate the scalar product of $|B\rangle_{fi}$ with the various massless closed string states. This gives the value of the 1-point correlators on a disk where the boundary conditions are determined by $f^i(v)$. As shown in [20], these couplings can be combined with a free propagator yielding the first two diagrams in Fig. 1; after a Fourier transformation from momentum to configuration space, these diagrams reproduce the solution in [36,37] at the linear level in the D1 and D5 charges.

We are now interested in considering more in detail the zero-mode structure of the boundary state, see Eqs. (3.14) and (3.15) of [20]. It follows that, even if both t and y are directions with Neumann boundary conditions, D-branes with a travelling wave can emit closed strings with a non-trivial momentum k along v provided that

$$k_u = 0, \quad k_v + \dot{f}^i k_i = 0, \quad (3.1)$$

where, as before, the dot indicates the derivative with the respect to v . Thus $k_v = 0$ is possible for special values of the k_i 's. If we limit ourselves to probes with zero momentum along v , then the string correlators contain always an integral over v and the smeared solution discussed in [20] is recovered. However if we test the D-brane configuration with a generic (localized) probe, then we obtain a v -dependent result for the string correlator and the original solution [36,37], without integrals over v , is obtained. Notice that these v -dependent 2-charge solutions cannot be dualized to the D1–D5 frame as it was done in [3]. The obstruction is clearly that for v -dependent geometries the shifts along y are not an isometry. This suggests that the only 2-charge microstates in the D1–D5 frame are those studied in [3] which always include a smearing over the S^1 .

It is possible to reach the same conclusion by working directly in the D1–D5 frame and following the microscopic approach used in this paper. In this language the D1–D5 microstates are built by starting from an unbound set of D-branes and switching on a vev for the open strings stretched between the D1 and the D5 branes. If we do not want to introduce any further charge or equivalently we wish to preserve the same number of supercharges of the unbound system, then all open string states introduced must have exactly zero momentum. Now it is clear also from this point of view why 2-charge configurations are always smeared along the S^1 : the boundary conditions appropriate for the basic D-brane constituents require momentum conservation both along v and u , and no open strings carry any momentum; thus the 1-point correlators are non-trivial only if the closed string probe is at zero momentum as well and then the results automatically include integrals over both common Neumann directions.

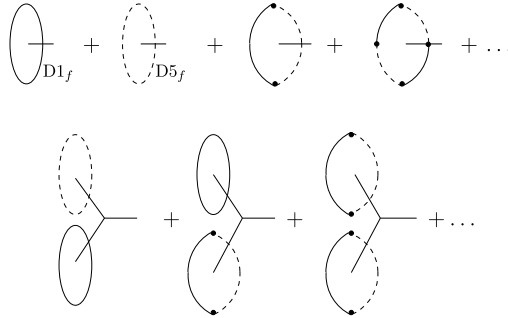


Fig. 1. A schematic picture of the diagrams relevant to the calculation of the gravitational backreaction of a microstate: the straight lines represent generic massless closed string states and the circles represent the boundary of the string world-sheet ending on the D-branes defining the microstate; the continuous (dashed) parts of the circles mean that the boundary conditions appropriate for a vibrating D1 brane (D5 brane) are imposed on the string coordinates, while the black dots on the transition between these two types of boundary conditions represent the insertion of a twisted open string state (i.e. an open string stretched between a D1 brane and D5 brane). The first line contains all diagrams with a single border, i.e. the contributions linear in the source. The second line contains the diagrams that are needed to reconstruct the non-linear solution; the vertices in the bulk follow from the non-linear part of the standard IIB supergravity equations.

We now wish to use localized probes to test the 3-charge systems studied in [21]: this means that we start from the unbound system mentioned at the beginning of this section, describing D1 and D5 branes oscillating with a common profile, and introduce a vev for the open strings stretched between the D1 and the D5 branes; then we probe the configuration with generic closed string states which have also a non-zero momentum k_v . As in [21], in this section we limit ourselves to the contributions of the first three diagrams in Fig. 1 and calculate explicitly the corresponding string amplitudes by using the RNS formalism. In particular the simplest class of microstates [3] corresponds to the configurations obtained by introducing a vev for the mixed D1–D5 open strings in the Ramond sector [19]. At the leading order, this condensate involves only two open string states (the black dots in Fig. 1) and so is described by a spinor bilinear which can be decomposed in a vector and a self-dual 3-form v_{IJK} living in the space orthogonal to the T^4 . We will focus exclusively on the contribution of the 3-form and, as done in [21], we also set to zero the components with just one (or all three) legs along the R^4 . So the non trivial part of v_{IJK} can be decomposed in two $SO(1, 1) \times SO(4)$ representations with opposite duality properties

$$v_{uij} = -\frac{1}{2}\epsilon_{ijkl}v_{ukl}, \quad v_{vij} = +\frac{1}{2}\epsilon_{ijkl}v_{vkl}. \quad (3.2)$$

The first guess is that the result will follow the same pattern discussed above in the D1–P (or D5–P) case and that the v -dependent backreaction would simply be the solution in Eq. (5.16)–(5.32) of [21], where all integrals over v (hidden in the definition of \mathcal{I}) are dropped. However it is not difficult to check that this guess cannot be correct, as the configuration just mentioned is not a solution of the supergravity equations even if we limit ourselves to the leading order in the charges and the condensate (3.2). It turns out that in the 3-charge case there are new contributions to the geometric backreaction that are invisible to delocalized probes. The origin of this is as follows: the string correlator is calculated in ten dimensions where the R^4 and the light-cone (u, v) directions share the feature of having the same boundary conditions (either Neumann or Dirichlet) on both types of branes. Then the correlators are more easily calculated in an $SO(1, 5)$ invariant way which keeps all these directions on the same footing. For instance this was done in

Eq. (4.14) of [21] for a generic NS–NS probe. At this level the generalization from a smeared to a localized probe involves just dropping the integral over v . However, the ansatz of the previous section is clearly not $SO(1, 5)$ covariant; then in order to identify the different supergravity fields we need to decompose the string result in $SO(1, 1) \times SO(4)$ representations. In doing this step in [21] it was assumed that the momentum of the closed string probe was entirely in the non-compact dimensions. So in order to read the new v -dependent geometry we have to reconsider this step. The starting point is Eq. (4.14) of that paper which describes the emission of NS–NS state from a disk with two twisted open string state (i.e. the third diagram of the first line of Fig. 1 in the NS–NS sector). By dropping the v integral we have

$$\mathcal{A}_{\text{NS}}^{\text{D1D5}} = -2\sqrt{2}\pi V_u e^{-ik_i f^i(v)} k^K \mathcal{G}^{IJ} ({}^t R)_J^M v_{IMK}, \quad (3.3)$$

where V_u is the infinite volume of the u direction, the uppercase indices run over v, u, x^i and R is (the zero-mode part of) the reflection matrix [20,21]²

$$R^\mu_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2|\dot{f}(v)|^2 & 1 & -2\dot{f}_i(v) & 0 \\ 2\dot{f}_i(v) & 0 & -\mathbb{1} & 0 \\ 0 & 0 & 0 & -\mathbb{1} \end{pmatrix}. \quad (3.4)$$

The matrix indices are ordered by putting first the light-cone coordinates v, u , then the R^4 indices i and finally the T^4 ones a . By decomposing (3.3) we obtain

$$\begin{aligned} \mathcal{A}_{\text{NS}}^{\text{D1D5}} = 2\sqrt{2}\pi V_u e^{-ik_i f^i(v)} k^l [& (\mathcal{G}^{uj} + \mathcal{G}^{ju}) v_{ujl} + (\mathcal{G}^{vj} + \mathcal{G}^{jv}) v_{vjl} \\ & - 2\eta^{uv} \mathcal{G}^{jv} |\dot{f}|^2 v_{ujl} - 2\mathcal{G}^{uv} \dot{f}_j v_{ujl} - 2\mathcal{G}^{vv} \dot{f}_j v_{vjl} + 2\eta^{uv} \mathcal{G}^{ij} \dot{f}_j v_{uil} \\ & + \eta^{uv} k_v [-2\mathcal{G}^{jv} \dot{f}_i v_{uji} + \mathcal{G}^{ij} v_{uij}], \end{aligned} \quad (3.5)$$

where \mathcal{G} is the polarization of a generic NS–NS state which needs to be further decomposed in the graviton, dilaton and B-field. If we set k_v to zero in (3.5) of course we recover Eq. (4.15) of [21].

A similar step has to be performed also when we use a closed string state in the RR sector as a probe. We start from Eq. (4.24) of [21], again without the integral over v

$$\mathcal{A}_R^{\text{D1D5}} = \frac{i\pi}{2} V_u e^{-ik \cdot f(v)} v_{IJK} (C \Gamma^{IJK})^{BA} (C^{-1} \mathbf{F} \mathcal{R} C^{-1})_{AB}, \quad (3.6)$$

where A, B and Γ 's are spinor indices and the Gamma matrices of $SO(1, 5)$, \mathbf{F} is a bispinor encoding the RR field strengths (see Eq. (4.23) of [21] for our conventions) and \mathcal{R} is the spinorial representation of the reflection matrix, i.e.

$$\mathcal{R} = \Gamma^{uv} - \dot{f}^i(v) \Gamma^{iv}. \quad (3.7)$$

Then by rewriting the field strengths in terms of the gauge potentials C we have

² In [20,21] the coordinates u and v were defined as in (2.1) but without the factor $1/\sqrt{2}$. In the following we adapt the results of those papers to our conventions for the light-cone coordinates: this is the reason why the form of the reflection matrices in (3.4) and (3.7) differ from the corresponding expressions in [21]. In the expressions (3.5) and (3.8) for the NS–NS and RR amplitudes the metric component η^{uv} equals -1 in the conventions of the present paper and -2 in the conventions of [20,21].

$$\begin{aligned}
A_{RR}^{D1D5} = & 4\pi V_u e^{-ik \cdot f(v)} \left[\eta^{uv} k^l C^{(0)} \dot{f}_j v_{ulj} + \eta^{uv} k^l C^{5678} \dot{f}_j v_{ulj} + k^i C^{uvjk} \dot{f}_k v_{uij} \right. \\
& + \frac{1}{2} k^k C^{uvij} \dot{f}_k v_{uij} + \frac{\eta^{uv}}{2} k_v C^{vijk} \dot{f}_k v_{uij} + \frac{\eta^{uv}}{2} k_v C^{ij} v_{uij} - k^i C^{uj} v_{uij} \\
& + k^i C^{vj} v_{vij} - \eta^{uv} k_v C^{vj} \dot{f}_l v_{ulj} - k^i C^{uv} \dot{f}_l v_{uli} + \frac{\eta^{uv}}{2} k^l C^{ij} \dot{f}_l v_{uij} \\
& \left. + \eta^{uv} k^j C^{li} \dot{f}_l v_{uij} \right]. \tag{3.8}
\end{aligned}$$

Notice the appearance, in the second term of the second line, of a contribution to the 4-form potential with three indices in the R^4 . Such a structure is absent in the smeared case where k_v is set to zero; this is the origin of the 3-form x_3 in the RR ansatz (2.8).

In our discussion so far we assumed that both the D1 and the D5 branes are wrapped once on the S^1 of radius R ; in this case the functions $f_i(v)$ describing their (common) shape in the R^4 are periodic under shifts $v \rightarrow v + 2\pi R$. However the most interesting configurations in the analysis of the black hole microstates involve D-branes with wrapping number w larger than one; then the profile f_i depends on the world-volume coordinate \hat{v} and has periodicity $2\pi w R$. Geometrically we can describe this situation by splitting the closed profile in w open segments $f_i^\alpha(v)$ with $\alpha = 1, \dots, w$, and then imposing the gluing conditions $f_i^\alpha(2\pi R) = f_i^{\alpha+1}(0)$ where $\alpha = w + 1$ is identified with $\alpha = 1$. We will not write the complete expression describing the boundary state corresponding to these multiply wrapped D-branes: since we consider the emission only of closed string states with zero winding number, we will treat each segment independently and sum over the individual results to obtain the coupling of the wrapped D-brane to the closed strings. This is sufficient for our purposes, since on the gravity side we work at the linearized level in the sources, i.e. we are interested in world-sheets with just one boundary. Another interesting issue that we will not analyze further is related to the special points in space–time where two of these segments intersect. The open strings living at these intersections will feel locally two D-branes with a relative rotation and boost; however the parameters of the transformations are tuned so as to always preserve supersymmetry (notice that the situation is different in the space-like [38] or time-like [39] cases). We will not make explicit use of these open string sectors. As a consistency check of our approach, we will show that the backreaction obtained by superimposing the contributions of each segment preserves four supercharges.

It is now straightforward to follow the procedure discussed in Section 4.3 of [21] and derive from (3.5) and (3.8) the configuration space results (3.9)–(3.20). As mentioned above, the label α indicates the different segments of the multiply wrapped profile (common to the D1 and D5 branes); also, in the expressions below, a sum over α , in each α -dependent term, is present even if it is not explicitly written

$$Z_1 = 1 + Q_1 \mathcal{I}^\alpha + \mathbf{v}_{ulk} \partial_l \mathcal{I}^\alpha \dot{f}_k^\alpha, \tag{3.9}$$

$$Z_2 = 1 + Q_5 \mathcal{I}^\alpha + \mathbf{v}_{ulk} \partial_l \mathcal{I}^\alpha \dot{f}_k^\alpha, \tag{3.10}$$

$$\mathcal{F} = -(Q_1 + Q_5) \mathcal{I}^\alpha |\dot{f}^\alpha|^2 - 2 \mathbf{v}_{ulk} \partial_l \mathcal{I}^\alpha \dot{f}_k^\alpha, \tag{3.11}$$

$$Z_4 = -\mathbf{v}_{ulk} \partial_l \mathcal{I}^\alpha \dot{f}_k^\alpha, \tag{3.12}$$

$$a_1 = Q_5 (\mathcal{I}^\alpha \dot{f}_i^\alpha + \dot{f}_k^\alpha \hat{\mathcal{I}}_{ki}^\alpha) dx^i + \mathbf{v}_{uli} \partial_l \mathcal{I}^\alpha |\dot{f}^\alpha|^2 dx^i, \tag{3.13}$$

$$\beta = \mathbf{v}_{uli} \partial_l \mathcal{I}^\alpha dx^i, \tag{3.14}$$

$$a_4 = [\mathbf{v}_{uil} \partial_l \mathcal{I}^\alpha |\dot{f}^\alpha|^2 + \mathbf{v}_{uil} \partial_v (\mathcal{I}^\alpha \dot{f}_i)] dx^i, \tag{3.15}$$

$$\omega = [(Q_1 + Q_5)\mathcal{I}^\alpha \dot{f}_i^\alpha + \mathbf{v}_{vli}\partial_l\mathcal{I}^\alpha + \mathbf{v}_{uli}\partial_l\mathcal{I}^\alpha |\dot{f}^\alpha|^2 + \mathbf{v}_{uli}\partial_v(\mathcal{I}^\alpha \dot{f}_l^\alpha)]dx^i, \quad (3.16)$$

$$\delta_2 = \left[\mathbf{v}_{uli}\partial_l\mathcal{I}^\alpha \dot{f}_j^\alpha - \frac{1}{2}\mathbf{v}_{uij}\partial_v\mathcal{I}^\alpha \right] dx^i \wedge dx^j, \quad (3.17)$$

$$\gamma_2 = \frac{1}{2}Q_5\hat{\mathcal{I}}_{ij}^\alpha dx^i \wedge dx^j - \mathbf{v}_{uli}\partial_l\mathcal{I}^\alpha \dot{f}_j^\alpha dx^i \wedge dx^j, \quad (3.18)$$

$$x_3 = \frac{1}{2}\mathbf{v}_{uij}\partial_v(\mathcal{I}^\alpha \dot{f}_k^\alpha) dx^i \wedge dx^j \wedge dx^k, \quad (3.19)$$

$$ds_4^2 = [\delta_{ij} + \mathbf{v}_{uli}\partial_l\mathcal{I}^\alpha \dot{f}_j^\alpha + \mathbf{v}_{ulj}\partial_l\mathcal{I}^\alpha \dot{f}_i^\alpha - \delta_{ij}\mathbf{v}_{ulk}\partial_l\mathcal{I}^\alpha \dot{f}_k^\alpha] dx^i dx^j, \quad (3.20)$$

where the Q_I 's indicate the D1 and D5 charges and \mathbf{v} is the open string condensate (3.2) after a constant rescaling

$$\mathbf{v}_{uij} = -\frac{2\sqrt{2}\kappa}{\pi V_4}v_{uij}, \quad \mathbf{v}_{vij} = -\frac{2\sqrt{2}\kappa}{\pi V_4}v_{vij}. \quad (3.21)$$

The function \mathcal{I}^α is harmonic and defines implicitly also the 2-form $\hat{\mathcal{I}}^\alpha$

$$\mathcal{I}^\alpha = \frac{1}{|x - f^\alpha(v)|^2}, \quad d\hat{\mathcal{I}}^\alpha = *_4 d\mathcal{I}^\alpha, \quad (3.22)$$

where the star is defined by using the flat R^4 metric and the differential d acts only on x^i and not on v . In the next section we check that the IIB background defined by the data above solves the linearized constraints following from supersymmetry and the equations of motion.

4. The linearized geometry

The linearized type IIB background obtained in the previous section has different 2-charge limits. We can switch off the condensate of D1–D5 strings \mathbf{v} and obtain the geometry corresponding to an unbound configuration of D1 and D5 branes or set to zero the geometric profile $f_i^\alpha(v)$ and obtain the 2-charge D1–D5 microstates. As mentioned before, these D1–D5 geometries are dual to the geometry of a vibrating string and their non-linear completion is known [3, 4,40]. In particular, the dependence of the full solution on the open string condensates can be expressed in terms of auxiliary periodic functions $g_i(v')$, whose moments are the vev's used in the world-sheet description of the previous section. For instance the condensate we considered (i.e. \mathbf{v}) is written in terms of $g_i(v')$ as follows [19]

$$\mathbf{v}_{tij} \sim \frac{1}{L'} \int_0^{L'} \dot{g}_i(v') g_j(v') dv', \quad (4.1)$$

where L' is the periodicity of $g_i(v')$ and the dot on g_i represents the derivative with respect to the auxiliary variable v' . Thus we can use the exact 2-charge D1–D5 solution to generalize the result (3.9)–(3.20). The idea is to promote all Q_I and \mathbf{v} -dependent terms to more general objects depending on $g_i(v')$ which should encode the exact-dependence on the open string condensates. This should account for the contribution of the diagrams that have more than two insertions of twisted open strings, see for instance the diagram at the end of the first line of Fig. 1.

We can implement the idea above by setting to zero the profile $f_i^\alpha(v)$. Then from the Lunin–Mathur [3,4] solution we can read the general dependence of the various object on the functions

$g_i(v')$ representing general twisted open string condensates. Z_1 and Z_2 , which for $f_i^\alpha(v) = 0$ are just harmonic functions centered in zero, become

$$Z_1^{\text{D1D5}} = 1 + \frac{Q_5}{L} \int_0^L \frac{dv' |\dot{g}(v')|^2}{|x - g(v')|^2}, \quad Z_2^{\text{D1D5}} = 1 + \frac{Q_5}{L} \int_0^L \frac{dv'}{|x - g(v')|^2}. \quad (4.2)$$

Similarly the general form of (3.14) and (3.16) (always at $f_i^\alpha(v) = 0$) is given by

$$\beta^{\text{D1D5}} = A^{\text{D1D5}} - B^{\text{D1D5}}, \quad \omega^{\text{D1D5}} = A^{\text{D1D5}} + B^{\text{D1D5}}, \quad (4.3)$$

with

$$A^{\text{D1D5}} = -\frac{Q_5}{L} \int_0^L \frac{dv' \dot{g}_i(v')}{|x - g(v')|^2} dx^i, \quad *_4 d B^{\text{D1D5}} = -d A^{\text{D1D5}}. \quad (4.4)$$

These quantities satisfy simple harmonic conditions

$$d *_4 d Z_1^{\text{D1D5}} = 0, \quad d *_4 d Z_2^{\text{D1D5}} = 0, \quad (4.5)$$

and the self-duality and anti-self-duality properties

$$*_4 d \beta^{\text{D1D5}} = d \beta^{\text{D1D5}}, \quad *_4 d \omega^{\text{D1D5}} = -d \omega^{\text{D1D5}}. \quad (4.6)$$

Thanks to (4.5) we can define a 2-form γ_2^{D1D5} satisfying

$$d \gamma_2^{\text{D1D5}} = *_4 d Z_2^{\text{D1D5}}. \quad (4.7)$$

Moreover, by possibly adding exact terms, we can also impose the gauge conditions

$$d *_4 \beta^{\text{D1D5}} = d *_4 \omega^{\text{D1D5}} = 0, \quad (4.8)$$

which are satisfied by the perturbative expressions of the previous section when $f_i^\alpha(v) = 0$. In this gauge it is possible to define a 2-form ζ^{D1D5} such that

$$d \zeta^{\text{D1D5}} = *_4 \beta^{\text{D1D5}}, \quad (4.9)$$

where ζ^{D1D5} itself is defined up to a gauge, which we can fix by imposing

$$\zeta^{\text{D1D5}} = - *_4 \zeta^{\text{D1D5}}. \quad (4.10)$$

Now the strategy is to include the dependence on the geometric profile $f_i^\alpha(v)$ as done in the previous section. First let us introduce the barred quantities which are related to the D1–D5 ones as follows

$$\begin{aligned} \bar{Z}_I^\alpha &= Z_I^{\text{D1D5}}(x - f^\alpha(v)), & \bar{\gamma}_2^\alpha &= \gamma_2^{\text{D1D5}}(x - f^\alpha(v)), \\ \bar{\beta}^\alpha &= \beta^{\text{D1D5}}(x - f^\alpha(v)), & \bar{\omega}^\alpha &= \omega^{\text{D1D5}}(x - f^\alpha(v)), \\ \bar{\zeta}^\alpha &= \zeta^{\text{D1D5}}(x - f^\alpha(v)). \end{aligned} \quad (4.11)$$

These new expressions still solve, of course, the same harmonic equations and duality conditions of the f -independent results written above

$$\begin{aligned} d *_4 d \bar{Z}_I^\alpha &= 0, & d \bar{\gamma}_2^\alpha &= *_4 d \bar{Z}_2^\alpha, & *_4 d \bar{\beta}^\alpha &= d \bar{\beta}^\alpha, & *_4 d \bar{\omega}^\alpha &= -d \bar{\omega}^\alpha, \\ d *_4 \bar{\beta}^\alpha &= d *_4 \bar{\omega}^\alpha = 0, & d \bar{\zeta}^\alpha &= *_4 \bar{\beta}^\alpha, & \bar{\zeta}^\alpha &= - *_4 \bar{\zeta}^\alpha. \end{aligned} \quad (4.12)$$

The v -dependence of the barred quantities is entirely implicit in their dependence on $f_i^\alpha(v)$, so that, for example,

$$\partial_v \bar{\beta}^\alpha = -\dot{f}_i^\alpha \partial_i \bar{\beta}^\alpha. \quad (4.13)$$

We will make repeatedly use of this identity in the following.

When only the first non-trivial term in the small $g_i(v')$ expansion of these results is kept, $\bar{\beta}^\alpha$ reduces to (3.14)

$$\bar{\beta}^\alpha = \mathbf{v}_{uli} \partial_l \mathcal{I}^\alpha dx^i, \quad (4.14)$$

and the explicit expression for $\bar{\zeta}^\alpha$ is

$$\bar{\zeta}^\alpha = -\frac{1}{2} \mathbf{v}_{uij} \mathcal{I}^\alpha dx^i \wedge dx^j, \quad (4.15)$$

which satisfies (4.12) thanks to (3.2). Then we can generalize (3.9)–(3.20) by redefining all the expressions appearing there as follows

$$Z_1 = \bar{Z}_1^\alpha + \bar{\beta}_k^\alpha \dot{f}_k^\alpha, \quad (4.16)$$

$$Z_2 = \bar{Z}_2^\alpha + \bar{\beta}_k^\alpha \dot{f}_k^\alpha, \quad (4.17)$$

$$\mathcal{F} = -(\bar{Z}_1^\alpha + \bar{Z}_2^\alpha - 2) |\dot{f}^\alpha|^2 - 2\bar{\omega}_k^\alpha \dot{f}_k^\alpha, \quad (4.18)$$

$$Z_4 = -\bar{\beta}_k^\alpha \dot{f}_k^\alpha, \quad (4.19)$$

$$a_1 = (\bar{Z}_2^\alpha - 1) \dot{f}_i^\alpha dx^i - \bar{\gamma}_{2ik}^\alpha \dot{f}_k^\alpha dx^i + \bar{\beta}^\alpha |\dot{f}^\alpha|^2, \quad (4.20)$$

$$\beta = \bar{\beta}^\alpha, \quad (4.21)$$

$$a_4 = -\bar{\beta}^\alpha |\dot{f}^\alpha|^2 + \partial_v (\bar{\zeta}_{ki}^\alpha \dot{f}_k^\alpha dx^i), \quad (4.22)$$

$$\omega = (\bar{Z}_1^\alpha + \bar{Z}_2^\alpha - 2) \dot{f}_i^\alpha dx^i + \bar{\omega}^\alpha + \bar{\beta}^\alpha |\dot{f}^\alpha|^2 - \partial_v (\bar{\zeta}_{ki}^\alpha \dot{f}_k^\alpha dx^i), \quad (4.23)$$

$$\delta_2 = \bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i + \partial_v \bar{\zeta}^\alpha, \quad (4.24)$$

$$\gamma_2 = \bar{\gamma}_2^\alpha - \bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i, \quad (4.25)$$

$$x_3 = -\partial_v (\bar{\zeta}^\alpha \wedge \dot{f}_i^\alpha dx^i), \quad (4.26)$$

$$ds_4^2 \equiv (\delta_{ij} + h_{ij}^{(1)}) dx^i dx^j = (\delta_{ij} + \bar{\beta}_i^\alpha \dot{f}_j^\alpha + \bar{\beta}_j^\alpha \dot{f}_i^\alpha - \delta_{ij} \bar{\beta}_k^\alpha \dot{f}_k^\alpha) dx^i dx^j. \quad (4.27)$$

As before we understood a sum over the label α indicating the contribution of each segment of the multiply wrapped D1 and D5 branes. The two-step procedure used to derive (4.16)–(4.27) seems justified from the world-sheet picture, where the data of the two profiles $g_i(v')$ and $f_i^\alpha(v)$ are encoded by completely different open string states. We will show that this more general configuration satisfies the supergravity equations just as a consequence of (4.12). Of course this implies that also the configuration of the previous section, where we kept only the first-order terms in the small $g_i(v')$ expansion, is a solution of the supergravity equations.

We leave most of the details of the check to [Appendix A](#) and collect in the main text only some results on the “almost hyper-Kähler” base metric. The first ingredient on which the whole solution is built is the 1-form β . Since we are working at the linearized order in β itself, Eq. (2.10) simplifies: the curved star \star_4 reduces to the flat one \ast_4 and the v -dependent differential D becomes the standard differential d in R^4 . Then the linearized (2.10) is just $d\beta = \ast_4 d\beta$ and it is a direct consequence of (4.12). The next step is to define the set of complex structures $J^{(A)}$ compatible with the 4D metric (4.27). In our case, these can be written in terms of $\bar{\beta}^\alpha$ and the trivial

complex structures $J_0^{(A)}$ appropriate for a flat R^4

$$\begin{aligned} J_0^{(1)} &= dx^1 \wedge dx^2 - dx^3 \wedge dx^4, \\ J_0^{(2)} &= dx^1 \wedge dx^3 + dx^2 \wedge dx^4, \\ J_0^{(3)} &= dx^1 \wedge dx^4 - dx^2 \wedge dx^3. \end{aligned} \quad (4.28)$$

Then at linear order in β we have

$$\begin{aligned} J^{(A)} &\equiv J_0^{(A)} + J_1^{(A)} \\ &= J_0^{(A)} - \frac{1}{2} [\dot{f}_i^\alpha (\bar{\beta}^\alpha \wedge J_0^{(A)})_{ijk} dx^j \wedge dx^k] \\ &= J_0^{(A)} - \bar{\beta}_i^\alpha \dot{f}_i^\alpha J_0^{(A)} - \bar{\beta}^\alpha \wedge J_{0ij}^{(A)} \dot{f}_j^\alpha dx^i. \end{aligned} \quad (4.29)$$

At first order in β the constraints (2.11) reduce to

$$\begin{aligned} J_0^{(A)} \wedge J_1^{(B)} + J_1^{(A)} \wedge J_0^{(B)} &= -h_{kk}^{(1)} \delta^{AB} dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4, \\ dJ_1^A &= \dot{\beta} \wedge J_0^A, \end{aligned} \quad (4.30)$$

where the trace of the first-order part of the metric is

$$h_{kk}^{(1)} = -2\bar{\beta}_k^\alpha \dot{f}_k^\alpha. \quad (4.31)$$

We leave to [Appendix A](#) the proof that (4.30) follows from (4.29), (4.28) and $d\beta = *_4 d\beta$. Let us conclude the discussion of the 4D base by noticing that the (linearized) ψ takes a very simple form

$$\psi = -\frac{1}{2} \sum_C \partial_v (\bar{\beta}_i^\alpha \dot{f}_j^\alpha) J_{0ij}^{(C)} J_0^{(C)} = -\frac{1}{2} \partial_v [\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i - *_4 (\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i)], \quad (4.32)$$

where, in order to get the second identity, we used

$$\sum_C J_{0ij}^{(C)} J_{0kl}^{(C)} = \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk} - \epsilon_{ijkl}. \quad (4.33)$$

5. A (partially) non-linear generalization

The results of the previous section contain all the genuinely stringy information on the superstrata that can be built by giving a (common) non-trivial profile to the D1 and D5 branes present in the Lunin–Mathur 2-charge microstates. In the language of perturbative string amplitudes this information captures the direct couplings between the supergravity fields and the D-branes forming the bound state, as depicted in the first line of [Fig. 1](#). In other words, the string calculation gives an explicit expression for the stress–energy side of Einstein’s equations and the “source-depending” side of all other supergravity equations. Clearly a string-theory derivation of the diagrams depicted in the second line of [Fig. 1](#) is very challenging, as it requires to deal with multi-loop open string diagrams (i.e. world-sheets with many boundaries). It is certainly easier to ignore all α' corrections and derive these non-linear terms by using supergravity: technically we have just to use the stringy results for the disk amplitudes as boundary conditions at large distances which fix the solution of the source-free supergravity equation.

At present, the only fully non-linear v -dependent solution representing a 3-charge microstate with a known CFT dual is the one discussed in [\[18\]](#). This example can be interpreted as a special

configuration where the functions $g_i(v')$ introduced in the previous section are directly determined by the geometric profile $f_i(v)$. The solution of [18] thus depends on only one independent profile and it represents a set of measure zero in the space of 3-charge microstates, that are expected to be parametrized by generic functions of two variables [25]. A first level of generalization consists in finding v -dependent solutions where the profiles $g_i(v')$ and $f_i(v)$ are unrelated. In this section we make a first step, by focusing on the class of superstrata considered in [26]. In that paper an exact solution carrying D1, D5 and P charges was constructed: that solution represents an unbound state of D1 and D5 branes oscillating according to a profile which has periodicity $2\pi R$ even if the D-branes are multiply wrapped. In the notations introduced in Section 3, we then have $f_i^\alpha(v) = f_i(v)$ for any α . The solution discussed in [26] is exact in the D1 and D5 charges and in the corresponding dipole charges, originating from the oscillation of the global D1 and D5 charges, but has no KK-monopole charge.

The aim of this section is to generalize the solution of [26]: we keep the full dependence on the D1 and D5 (dipole) charges, but we also wish to include at first order the effects of the KK-monopole dipole charge and the angular momentum; this will also turn the configuration into a real bound state. In the diagrammatic language of Fig. 1, this solution captures also some of the non-linear terms depicted in the second line: in particular we need to include all diagrams with at most one boundary which can contribute to β and ω at the linear level, but with an arbitrary number of other type of boundaries. However, as mentioned above, in order to capture these contributions we will not follow the perturbative approach, but a trick closely related to the approach of [41], which was also used by [36,37] to generate the gravity solutions corresponding to an oscillating string. The key point is the following: when $f_i^\alpha(v) = f_i(v)$, the dependence on $f_i(v)$ of the 4D base metric ds_4^2 in (4.27) can be completely absorbed in the change of coordinates $x^i \rightarrow x^i + f^i(v)$. This suggests that, when the solution (4.16)–(4.27) depends on a common profile, it takes a particularly simple form in the new coordinate system. The result obtained after the shift can again be expressed in terms of the ansatz discussed in Section 2 and we obtain the following simple set of geometric data

$$Z'_1 = Z_1^{\text{D1D5}}, \quad (5.1)$$

$$Z'_2 = Z_2^{\text{D1D5}}, \quad (5.2)$$

$$\mathcal{F}' = -|\dot{f}|^2 - 2\zeta_{lk}^{\text{D1D5}} \ddot{f}_l \dot{f}_k, \quad (5.3)$$

$$Z'_4 = -\beta_k^{\text{D1D5}} \dot{f}_k, \quad (5.4)$$

$$a'_1 = -\dot{f}_i dx^i, \quad (5.5)$$

$$\beta' = \beta^{\text{D1D5}}, \quad (5.6)$$

$$a'_4 = \zeta_{ki}^{\text{D1D5}} \ddot{f}_k dx^i, \quad (5.7)$$

$$\begin{aligned} \omega' = & -\dot{f}_i dx^i + \omega^{\text{D1D5}} + \beta^{\text{D1D5}} |\dot{f}|^2 - \beta_k^{\text{D1D5}} \dot{f}_k \dot{f}_i dx^i \\ & + \partial_l \zeta_{ki}^{\text{D1D5}} \dot{f}_l \dot{f}_k dx^i - \zeta_{ki}^{\text{D1D5}} \ddot{f}_k dx^i, \end{aligned} \quad (5.8)$$

$$\delta'_2 = \beta^{\text{D1D5}} \wedge \dot{f}_i dx^i - \partial_k \zeta^{\text{D1D5}} \dot{f}_k, \quad (5.9)$$

$$\gamma'_2 = \gamma_2^{\text{D1D5}} - \beta^{\text{D1D5}} \wedge \dot{f}_i dx^i, \quad (5.10)$$

$$x'_3 = \partial_k \zeta^{\text{D1D5}} \dot{f}_k \wedge \dot{f}_i dx^i - \zeta^{\text{D1D5}} \wedge \ddot{f}_i dx^i, \quad (5.11)$$

$$ds_4'^2 = dx^i dx^i. \quad (5.12)$$

In [Appendix B](#) we give the explicit expression of the relation between the new and the old geometric data induced by an $f_i(v)$ -dependent shift of the coordinates for the R^4 . Clearly the supergravity configuration obtained in this way is guaranteed, by construction, to solve the equations of motion in the same approximation used in the previous section, i.e. at first order in a simultaneous expansion in both the D1 and D5 charges and the KK-monopole dipole charge (to which β^{D1D5} , and therefore ζ^{D1D5} , and ω^{D1D5} are proportional).

The situation is actually a bit better: a slightly modified ansatz actually solves the supergravity equations *exactly* in the D1 and D5 (dipole) charges, and at first order in the KK-monopole dipole charge. The basic reason for this drastic simplification is twofold: first in these coordinates the R^4 metric is flat [\(5.12\)](#), and second the combinations Θ'_1 and Θ'_2 vanish

$$\Theta'_1 = \Theta'_2 = 0, \quad (5.13)$$

as can be easily verified by using the definitions [\(2.17\)](#). For instance

$$\Theta'_1 = \partial_v(\gamma'_2 - \beta' \wedge a'_1) = \partial_v(\gamma_2^{\text{D1D5}} - \beta^{\text{D1D5}} \wedge \dot{f}_i dx^i + \beta^{\text{D1D5}} \wedge \dot{f}_i dx^i) = 0, \quad (5.14)$$

which follows from the fact that a'_1 is constant and that all quantities with the “D1D5” superscript are independent of f_i and thus of v . Of course the presence of terms that do not vanish at large $|x^i|$ in a'_1 , ω' and \mathcal{F}' means that this solution is not asymptotically Minkowski and thus this coordinate frame is not the most suited to study the physical properties of the microstate geometry. However the frame where the metric $ds_4'^2$ is flat is the perfect setup to study the non-linear corrections induced by the supergravity equations. So we will use this approach as a way of generating non-linear solutions and then transform them back with the opposite shift $x^i \rightarrow x^i - f^i(v)$ to asymptotically flat geometries which are directly relevant to the problem of studying the 3-charge microstates.

Let us now discuss how [\(5.1\)–\(5.12\)](#) need to be modified in order to provide a solution at all orders in Q_1 and Q_5 , but only at the linearized level in β^{D1D5} and ω^{D1D5} , which capture the presence of a KK-monopole dipole charge. Actually, the only equation³ that receives corrections at our level of approximation is the one for x'_3 . It can be shown [\[30\]](#) that Eq. [\(2.21\)](#) should be generalized as follows

$$dx'_3 = da'_4 \wedge \gamma_2 - a'_1 \wedge d\delta'_2 + Z'_2 *_4 \partial_v Z'_4, \quad (5.15)$$

where we now have to consider the factor of Z_2 in the last term as it contains the dependence on Q_5 which we wish to keep exact; also we need to include the first two terms because, after the shift, a_1 is constant and the term γ_2^{D1D5} in γ_2 is independent of the KK-monopole dipole charge. The solution of this equation, at linear order in the KK-monopole charge, is

$$x'_3 = \partial_k \zeta^{\text{D1D5}} \dot{f}_k \wedge \dot{f}_i dx^i - Z_2^{\text{D1D5}} \zeta^{\text{D1D5}} \wedge \ddot{f}_i dx^i + \zeta_{ki}^{\text{D1D5}} \ddot{f}_k dx^i \wedge \gamma_2^{\text{D1D5}}. \quad (5.16)$$

Thus summarizing, the configuration specified by the data [\(5.1\)–\(5.12\)](#), where x_3 is substituted with the result above, solves the supergravity equations at linear order in β^{D1D5} and ω^{D1D5} .

We leave the explicit check of this statement to a forthcoming publication. Here we can support it by showing how the new solution looks in the original frame, where the 10D metric is asymptotically flat. Thus we can use the formulae of [Appendix B](#) and perform the coordinate

³ In principle also Eq. [\(2.23\)](#) for \mathcal{F} receives non-linear corrections in the D1 and D5 charges, as can be seen for example from Eq. [\(4.12\)](#) of [\[26\]](#). However, when $\Theta_I = 0$, these corrections involve the v -derivatives of Z_I , and thus vanish for the ansatz [\(5.1\)–\(5.12\)](#).

shift $x^i \rightarrow x^i - f^i(v)$, so as to go back to the frame where the solution is asymptotically flat. However this time we keep terms of all orders in Q_1 and Q_5 and linearize the change of variables only in the KK-monopole charge. We thus arrive at the solution specified by the following data

$$Z_1 = \bar{Z}_1(1 + \bar{\beta}_k \dot{f}_k), \quad (5.17)$$

$$Z_2 = \bar{Z}_2(1 + \bar{\beta}_k \dot{f}_k), \quad (5.18)$$

$$\mathcal{F} = -(\bar{Z}_1 \bar{Z}_2 - 1)(1 + \bar{\beta}_k \dot{f}_k)|\dot{f}|^2 - 2\bar{\omega}_k \dot{f}_k, \quad (5.19)$$

$$Z_4 = -\bar{\beta}_k \dot{f}_k, \quad (5.20)$$

$$a_1 = (\bar{Z}_2 - 1)\dot{f}_i dx^i - \bar{\gamma}_{2ik} \dot{f}_k dx^i + \bar{Z}_2 \bar{\beta} |\dot{f}|^2, \quad (5.21)$$

$$\beta = \bar{\beta}, \quad (5.22)$$

$$a_4 = -\bar{\beta} |\dot{f}|^2 + \partial_v (\bar{\zeta}_{ki} \dot{f}_k dx^i), \quad (5.23)$$

$$\omega = (\bar{Z}_1 \bar{Z}_2 - 1)(1 + \bar{\beta}_k \dot{f}_k) \dot{f}_i dx^i + \bar{\omega} + \bar{Z}_1 \bar{Z}_2 \bar{\beta} |\dot{f}|^2 - \partial_v (\bar{\zeta}_{ki} \dot{f}_k dx^i), \quad (5.24)$$

$$\delta_2 = \bar{\beta} \wedge \dot{f}_i dx^i + \partial_v \bar{\zeta}, \quad (5.25)$$

$$\gamma_2 = \bar{\gamma}_2 - \bar{\beta} \wedge (\dot{f}_i dx^i - \bar{\gamma}_{2ij} \dot{f}_i dx^j), \quad (5.26)$$

$$x_3 = -\partial_v \bar{\zeta} \wedge \dot{f}_i dx^i - \bar{Z}_2 \bar{\zeta} \dot{f}_i dx^i + (\bar{\zeta}_{ki} \dot{f}_k dx^i + \bar{\beta}_k \dot{f}_k \dot{f}_i dx^i) \wedge \bar{\gamma}_2, \quad (5.27)$$

$$ds_4^2 = (\delta_{ij} + \bar{\beta}_i \dot{f}_j + \bar{\beta}_j \dot{f}_i - \delta_{ij} \bar{\beta}_k \dot{f}_k) dx^i dx^j, \quad (5.28)$$

where we used the same conventions of the previous section, but we dropped all superscript α , since we are now working under the assumption $f_i^\alpha(v) = f_i(v)$. It is interesting to compare this result with the solution of [26]. Even if our solution falls into an enlarged ansatz, where all fields of type IIB supergravity are non-trivial, the extra fields, which are encoded in Z_4 , a_4 , δ_2 and x_3 , arise from the combined effect of having both a KK-monopole charge and an oscillating profile. Hence, when β^{D1D5} and ω^{D1D5} are set to zero our solution should reduce to the result of [26]. This is indeed the case, as it can be checked by comparing (5.19) and (5.24) with Eqs. (4.11) and (4.13) of [26], when the arbitrary parameters (c_1 , c_2 and the harmonic function H_3) in those equations are chosen appropriately. The geometric data given above provide a generalization of the result of [26] which includes the first corrections in β^{D1D5} and ω^{D1D5} .

6. Conclusions

In this paper we take the first steps towards the construction of supergravity solutions describing the class of bound states carrying D1, D5 and P charges introduced in [25] with the name of superstrata. The example of superstrata we construct carry four dipole charges corresponding to D1 and D5 branes, to an F1-string, and to a KK-monopole. We have obtained the geometries via successive levels of approximation. First we considered the solution as an expansion around flat space and for the most part we discarded terms of order higher than the first in this expansion. This corresponds to the solution (3.9)–(3.20), that results from summing the first three types of string diagrams in Fig. 1: it is a linearized solution in which, moreover, the linear terms receive contributions only from a finite number (zero or two) of insertions of the string condensate associated with the open strings stretching between D1 and D5 branes.

Exploiting the fact that the D1–D5 solution, that resums arbitrary numbers of D1–D5 condensate insertions, is known [3], and that the dependence on the oscillation profile $f_i(v)$ can be exactly computed in the WCFT, we infer the geometry (4.16)–(4.27), that gives the complete linearized solution for a superstratum: this solution should contain the information of all the

string disk diagrams and, together with the non-linear information encoded in the supergravity equations, should allow to reconstruct the full exact geometry.

We make a first step towards the non-linear completion of the solution in the particular case in which all the strands of the multiply wound D1–D5 string are described by the same oscillation profile, i.e. when $f_i^\alpha(v) = f_i(v)$, $\forall \alpha$. In this case one can apply a trick analogous to the one used in [41,36,37] and move to a coordinate frame where the equations simplify, though the solution ceases to be explicitly asymptotically Minkowski. Transforming back to an asymptotically flat frame, we arrive at the solution (5.17)–(5.28), that solves the equations at all orders in the D1 and D5 charges, but only at first order in the KK-monopole dipole charge; this solution represents the first-order deformation of the solution of [26] upon the addition of the fourth dipole charge.

Our work opens the way to several future developments. The extension of our result to an exact solution of supergravity will not only represent a technical improvement but it will provide important physical insights on the nature of black hole microstates: it will allow us to probe a larger class of 3-charge microstate geometries at scales where a classical horizon is expected to form, and to verify their smoothness or their eventual singularity structure.

The solutions we find are v -dependent geometries that contain more fields (making up one more 6D tensor multiplet) than the ones present in the ansatz of [26]. The conditions for supersymmetry in this enlarged v -dependent setting are not known: to aim at a non-linear extension of our results a first necessary step is thus the derivation of the appropriate set of supergravity equations. Work in this directions is in progress [30].

With the supergravity equations at hand, and exploiting the trick introduced in Section 5, we think that a fully non-linear completion of the solution (5.17)–(5.28), describing a superstratum where the various strands oscillate with the same profile, should be within reach.

For a generic superstratum, described by strands oscillating with independent profiles, the problem seems much more intricate, and potentially interesting.⁴ In particular there does not seem to exist a coordinate frame that trivializes the 4D base metric given in (4.27). One is thus faced with the highly non-linear problem of finding an “almost hyper-Kähler” metric and a 1-form β that solve the constraints (2.10)–(2.11) and reduce to (4.27) and (4.21) at the linear level. It was however shown in [26] that this is the only intrinsically non-linear part of the problem: the remaining equations, if solved in the right order, reduce to a sequence of linear equations.

Finally we note that we landed onto a supergravity ansatz that generalizes the one of [26] by starting from the simplest world-sheet string configuration describing a bound state of D1–D5–P charges. In particular we decided to switch on only the components of the D1–D5 string condensate associated with the 2-charge microstates of [3], but general condensates are possible, corresponding to the microstates of [40]. Moreover, we took the condensate to be v -independent, so that momentum is entirely carried by the oscillation profile $f_i(v)$. It is conceivable (and some preliminary computations support this possibility) that by starting from a more general world-sheet setup and by fine-tuning the various ingredients at our disposal, one could engineer a microscopic worksheet configuration that only sources the fields present in the restricted ansatz of [26]. Most likely, having a simpler supergravity ansatz should contribute to make the task of constructing a fully non-linear solution more tractable. The price to pay for this simplification is

⁴ Already for solutions with no KK-monopole dipole charge it was noted in [27] that new shape–shape interaction terms arise in generic superstrata with unequal strands.

that the microscopic D-brane configuration will be more complicated and thus the derivation of the linearized solution from string amplitudes will require more effort.

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Appendix A. Checking the linearized equations of motion

We will explicitly verify that the geometry given in (4.16)–(4.27) solves the linearized equations of motion as a consequence of (4.12).

We already noted in the text that β trivially solves its equation (2.10) at linear order. Let us now look at the equations for the 4D base ds_4^2 : the linearized equations are given in (4.30). The first is an algebraic constraint that can be verified starting from the explicit form of $J_1^{(A)}$ given in (4.29):

$$\begin{aligned}
 \frac{1}{2}(J_1^{(A)} \wedge J_0^{(B)} + J_1^{(B)} \wedge J_0^{(A)}) &\equiv J_1^{((A)} \wedge J_0^{(B))} \\
 &= -\bar{\beta}_i^\alpha \dot{f}_i^\alpha J_0^{(A)} \wedge J_0^{(B)} - \bar{\beta}^\alpha \wedge J_{0ij}^{((A)} \dot{f}_j^\alpha dx^i \wedge J_0^{(B))} \\
 &= 2\bar{\beta}_i^\alpha \dot{f}_i^\alpha \delta^{AB} d^4x - \frac{1}{2}\epsilon_{ijkl}\bar{\beta}_i^\alpha J_{0jm}^{((A)} J_{0kl}^{(B))} \dot{f}_l^\alpha d^4x \\
 &= 2\bar{\beta}_i^\alpha \dot{f}_i^\alpha \delta^{AB} d^4x + \bar{\beta}_i^\alpha J_{0jm}^{((A)} J_{0ij}^{(B))} \dot{f}_m^\alpha d^4x \\
 &= 2\bar{\beta}_i^\alpha \dot{f}_i^\alpha \delta^{AB} d^4x - \bar{\beta}_i^\alpha \dot{f}_i^\alpha \delta^{AB} d^4x \\
 &= \bar{\beta}_i^\alpha \dot{f}_i^\alpha \delta^{AB} d^4x = -\frac{1}{2}h_{kk}^{(1)}\delta^{AB} d^4x.
 \end{aligned} \tag{A.1}$$

Here we have introduced the short-hand notation

$$d^4x \equiv dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4, \tag{A.2}$$

and we have used the anti-self-duality of $J_0^{(A)}$ and the property

$$J_{0ik}^{(A)} J_{0kj}^{(B)} = \epsilon^{ABC} J_{0ij}^{(C)} - \delta^{AB} \delta_{ij}, \tag{A.3}$$

which is the zeroth-order version of (2.13). The last step follows from (4.31).

The differential constraint in (4.30) can be proved as follows:

$$\begin{aligned}
 dJ_1^{(A)} - \dot{\beta} \wedge J_0^{(A)} &= -d(\bar{\beta}_i^\alpha \dot{f}_i^\alpha) J_0^{(A)} - d\bar{\beta}^\alpha \wedge J_{0ij}^{(A)} \dot{f}_j^\alpha dx^i + \dot{f}_i^\alpha \partial_i \bar{\beta}^\alpha \wedge J_0^{(A)} \\
 &= dx^i \dot{f}_j^\alpha \wedge [(\partial_j \bar{\beta}_i^\alpha - \partial_i \bar{\beta}_j^\alpha) J_0^{(A)} - d\bar{\beta}^\alpha J_{0ij}^{(A)}] \\
 &= -dx^i \dot{f}_j^\alpha \wedge [(d\bar{\beta}^\alpha)_{ij} J_0^{(A)} + d\bar{\beta}^\alpha J_{0ij}^{(A)}],
 \end{aligned} \tag{A.4}$$

where we used (4.13). To see that this is zero, let us take its Hodge dual

$$*_4(dJ_1^{(A)} - \dot{\beta} \wedge J_0^{(A)}) = -\frac{1}{2} dx^i \epsilon_{ijkl} [(d\bar{\beta}^\alpha)_{jm} J_{0kl}^{(A)} + (d\bar{\beta}^\alpha)_{jk} J_{0lm}^{(A)}] \dot{f}_m^\alpha. \tag{A.5}$$

If we use the anti-self-duality of $J_0^{(A)}$ in the first term of the r.h.s. and the self-duality of $d\bar{\beta}^\alpha$ in the second term, we find

$$*_4(dJ_1^{(A)} - \dot{\beta} \wedge J_0^{(A)}) = dx^i [(d\bar{\beta}^\alpha)_{jm} J_{0ij}^{(A)} - (d\bar{\beta}^\alpha)_{il} J_{0lm}^{(A)}] \dot{f}_m^\alpha. \quad (\text{A.6})$$

If, vice versa, we use the self-duality of $d\bar{\beta}^\alpha$ in the first term of the r.h.s. and the anti-self-duality of $J_0^{(A)}$ in the second term, we find

$$\begin{aligned} *_4(dJ_1^{(A)} - \dot{\beta} \wedge J_0^{(A)}) &= -\frac{1}{4} dx^i \epsilon_{ijkl} [\epsilon_{jmab} (d\bar{\beta}^\alpha)_{ab} J_{0kl}^{(A)} - (d\bar{\beta}^\alpha)_{jk} \epsilon_{lmab} J_{0ab}^{(A)}] \dot{f}_m^\alpha \\ &= \frac{1}{2} dx^i (d\bar{\beta}^\alpha)_{kl} J_{0kl}^{(A)} \dot{f}_i^\alpha + dx^i (d\bar{\beta}^\alpha)_{li} J_{0kl}^{(A)} \dot{f}_k^\alpha \\ &\quad - \frac{1}{2} dx^i (d\bar{\beta}^\alpha)_{jk} J_{0jk}^{(A)} \dot{f}_i^\alpha - dx^i (d\bar{\beta}^\alpha)_{jk} J_{0ki}^{(A)} \dot{f}_j^\alpha \\ &= dx^i [(d\bar{\beta}^\alpha)_{il} J_{0lm}^{(A)} - (d\bar{\beta}^\alpha)_{jm} J_{0ij}^{(A)}] \dot{f}_m^\alpha. \end{aligned} \quad (\text{A.7})$$

If we compare (A.6) and (A.7) we see that the expressions on the r.h.s. are equal and opposite and hence vanish: the second equation in (4.30) is thus satisfied.

Let us now pass to the equation for a_1 : at linear order the definition of Θ_1 in (2.17) becomes

$$\begin{aligned} \Theta_1 &= da_1 + \dot{\gamma}_2 \\ &= d(\bar{Z}_2^\alpha \dot{f}_i^\alpha dx^i - \bar{\gamma}_{2ik}^\alpha \dot{f}_k^\alpha dx^i) + \partial_v \bar{\gamma}_2^\alpha + d\bar{\beta}^\alpha |\dot{f}^\alpha|^2 - \partial_v (\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i), \end{aligned} \quad (\text{A.8})$$

where the second expression follows from the ansatz (4.16)–(4.27). Using (the dual of) the identity

$$\begin{aligned} *_4d(-\bar{\gamma}_{2ik}^\alpha \dot{f}_k^\alpha dx^i) &= -\frac{1}{2} dx^i \wedge dx^j \epsilon_{ijkl} \partial_k \bar{\gamma}_{2lm}^\alpha \dot{f}_m^\alpha \\ &= -\frac{1}{4} dx^i \wedge dx^j \epsilon_{ijkl} \epsilon_{klmp} \partial_p \bar{Z}_2^\alpha \dot{f}_m^\alpha + \frac{1}{4} dx^i \wedge dx^j \epsilon_{ijkl} \partial_m \bar{\gamma}_{2kl}^\alpha \dot{f}_m^\alpha \\ &= d(\bar{Z}_2^\alpha \dot{f}_i^\alpha dx^i) - \partial_v (*_4 \bar{\gamma}_2^\alpha), \end{aligned} \quad (\text{A.9})$$

that descends from the second relation in (4.12), we can rewrite

$$\Theta_1 = d(\bar{Z}_2^\alpha \dot{f}_i^\alpha dx^i) + *_4d(\bar{Z}_2^\alpha \dot{f}_i^\alpha dx^i) + d\bar{\beta}^\alpha |\dot{f}^\alpha|^2 - \partial_v (\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i). \quad (\text{A.10})$$

The linearized version of the a_1 equation (2.18) is

$$*_4(\Theta_1 - \psi) = \Theta_1 - \psi. \quad (\text{A.11})$$

Using the expression for ψ given in (4.32) and the one for Θ_1 derived above one finds

$$\begin{aligned} \Theta_1 - \psi &= d(\bar{Z}_2^\alpha \dot{f}_i^\alpha dx^i) + *_4d(\bar{Z}_2^\alpha \dot{f}_i^\alpha dx^i) + d\bar{\beta}^\alpha |\dot{f}^\alpha|^2 \\ &\quad - \frac{1}{2} \partial_v [\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i + *_4(\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i)], \end{aligned} \quad (\text{A.12})$$

which shows explicitly that $\Theta_1 - \psi$ is self-dual, as required by (A.11).

From the first of (2.16), the linearized version of the Z_2 equation is

$$d\gamma_2 = *_4(dZ_2 + \dot{\beta}). \quad (\text{A.13})$$

From the ansatz (4.16)–(4.27), and the identity (4.13), one finds

$$dZ_2 + \dot{\beta} = d\bar{Z}_2^\alpha - dx^i (\partial_i \bar{\beta}_k^\alpha - \partial_k \bar{\beta}_i^\alpha) \dot{f}_k^\alpha; \quad (\text{A.14})$$

hence, making use of the second and third relation in (4.12), one has

$$\begin{aligned} *_4(dZ_2 + \dot{\beta}) &= *_4d\bar{Z}_2^\alpha - \frac{1}{3!} dx^i \wedge dx^j \wedge dx^k \epsilon_{ijkl} (d\bar{\beta}^\alpha)_{lm} \dot{f}_m^\alpha \\ &= -d\bar{\gamma}_2^\alpha - \frac{1}{23!} dx^i \wedge dx^j \wedge dx^k \epsilon_{ijkl} \epsilon_{lmnpq} (d\bar{\beta}^\alpha)_{pq} \dot{f}_m^\alpha \\ &= -d\bar{\gamma}_2^\alpha + d\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i = -d\gamma_2, \end{aligned} \quad (\text{A.15})$$

where in the last step we have used (4.25). We have thus obtained the Hodge dual of (A.13). As we have already explained, (A.13) implies the linearized version of the Z_2 equation in (2.19).

Analogously, to prove the Z_1 equation in (2.19) it is easier to show that there exists a γ_1 that solves the second equation in (2.16), which to linear order is

$$d\gamma_1 = *_4(dZ_1 + \dot{\beta}). \quad (\text{A.16})$$

If one defines

$$\gamma_1 = \bar{\gamma}_1^\alpha - \bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i, \quad (\text{A.17})$$

where

$$d\bar{\gamma}_1^\alpha = *_4d\bar{Z}_1^\alpha, \quad (\text{A.18})$$

(such a $\bar{\gamma}_1^\alpha$ exists thanks to the first of (4.12)) the proof proceeds as for Z_2 .

Let us now come to the equations for the new multiplet, Z_4, a_4, δ_2, x_3 . To verify the first equation in (2.20), let us start from (4.19) and compute

$$\begin{aligned} *_4dZ_4 &= -\frac{1}{3!} dx^i \wedge dx^j \wedge dx^k \epsilon_{ijkl} \partial_l \bar{\beta}_m^\alpha \dot{f}_m^\alpha \\ &= -\frac{1}{3!} dx^i \wedge dx^j \wedge dx^k \epsilon_{ijkl} (d\bar{\beta}^\alpha)_{lm} \dot{f}_m^\alpha - \frac{1}{3!} dx^i \wedge dx^j \wedge dx^k \epsilon_{ijkl} \partial_m \bar{\beta}_l^\alpha \dot{f}_m^\alpha \\ &= d\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i + \partial_v (*_4\bar{\beta}^\alpha) = d(\bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i + \partial_v \bar{\zeta}^\alpha) = d\delta_2, \end{aligned} \quad (\text{A.19})$$

where in the intermediate steps we have used again the self-duality of $d\bar{\beta}$ and the definition of $\bar{\zeta}$ in (4.12) and the relation (4.13), and in the last step we have compared with the form of δ_2 given in (4.24).

To prove the a_4 equation (the second equation in (2.20)), we need the identity

$$*_4d(\bar{\zeta}_{ki}^\alpha \dot{f}_k^\alpha dx^i) = \bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i + \partial_v \bar{\zeta}^\alpha, \quad (\text{A.20})$$

which can be shown as follows

$$\begin{aligned} *_4d(\bar{\zeta}_{ki}^\alpha \dot{f}_k^\alpha dx^i) &= \frac{1}{2} dx^i \wedge dx^j \epsilon_{ijkl} \partial_k \bar{\zeta}_{ml}^\alpha \dot{f}_m^\alpha \\ &= \frac{1}{4} dx^i \wedge dx^j \epsilon_{ijkl} \epsilon_{kmnp} \bar{\beta}_p^\alpha \dot{f}_m^\alpha + \frac{1}{4} dx^i \wedge dx^j \epsilon_{ijkl} \partial_m \bar{\zeta}_{kl}^\alpha \dot{f}_m^\alpha \\ &= \bar{\beta}^\alpha \wedge \dot{f}_i^\alpha dx^i + \partial_v \bar{\zeta}^\alpha, \end{aligned} \quad (\text{A.21})$$

where we have used the defining properties of $\bar{\zeta}^\alpha$ (the last two identities in (4.12)) and the analogue of (4.13) for $\bar{\zeta}^\alpha$. The ansatz (4.16)–(4.27) gives

$$da_4 + \dot{\delta}_2 = -d\bar{\beta}^\alpha |f_i^\alpha|^2 + \partial_v [d(\bar{\zeta}_{ki}^\alpha f_k^\alpha dx^i) + \bar{\beta}^\alpha \wedge f_i^\alpha dx^i + \partial_v \bar{\zeta}^\alpha]; \quad (\text{A.22})$$

the identity (A.20) shows that the quantity in square brackets on the r.h.s. of the above expression is self-dual, while the self-duality of $d\bar{\beta}^\alpha$ guarantees that the first term on the r.h.s. is self-dual. Hence the second relation in (2.20) is satisfied.

To verify the linearized x_3 equation (2.21) let us compute

$$dx_3 = -\partial_v d(\bar{\zeta}^\alpha \wedge f_i^\alpha dx^i) = -\frac{1}{2} \epsilon_{ijkl} \partial_v (\partial_i \bar{\zeta}_{jk}^\alpha f_l^\alpha) d^4x = -\partial_v (\bar{\beta}_i^\alpha f_i^\alpha) d^4x, \quad (\text{A.23})$$

where in the last step we have used that $d\bar{\zeta}^\alpha = *_4 \bar{\beta}^\alpha$; this is indeed equal to

$$*_4 \dot{Z}_4 = - *_4 \partial_v (\bar{\beta}_i^\alpha f_i^\alpha). \quad (\text{A.24})$$

Verifying the ω equation (2.22) amounts to show that the Θ_2 derived from (2.22) can be written as in second of (2.17), for some 1-form a_2 . At the linear level one should have that

$$\Theta_2 = da_2 + \gamma_1, \quad (\text{A.25})$$

where γ_1 is given in (A.17). Using the expressions from the ansatz (4.16)–(4.27), together with the self-duality of $d\bar{\beta}^\alpha$ and the anti-self-duality of $d\bar{\omega}^\alpha$, we find

$$\begin{aligned} d\omega + *_4 d\omega &= (d\bar{Z}_1^\alpha + d\bar{Z}_2^\alpha) \wedge f_i^\alpha dx^i + *_4 [(d\bar{Z}_1^\alpha + d\bar{Z}_2^\alpha) \wedge f_i^\alpha dx^i] + 2d\bar{\beta}^\alpha |f_i^\alpha|^2 \\ &\quad - \partial_v [d(\bar{\zeta}_{ki}^\alpha f_k^\alpha dx^i) + *_4 d(\bar{\zeta}_{ki}^\alpha f_k^\alpha dx^i)]. \end{aligned} \quad (\text{A.26})$$

The terms in the second line can be simplified with the help of the identity (A.20) and the fact that $\bar{\zeta}$ is anti-self-dual, obtaining

$$\begin{aligned} d\omega + *_4 d\omega &= (d\bar{Z}_1^\alpha + d\bar{Z}_2^\alpha) \wedge f_i^\alpha dx^i + *_4 [(d\bar{Z}_1^\alpha + d\bar{Z}_2^\alpha) \wedge f_i^\alpha dx^i] + 2d\bar{\beta}^\alpha |f_i^\alpha|^2 \\ &\quad - \partial_v [\bar{\beta}^\alpha \wedge f_i^\alpha dx^i + *_4 d(\bar{\beta}^\alpha \wedge f_i^\alpha dx^i)]. \end{aligned} \quad (\text{A.27})$$

Subtracting from $d\omega + *_4 d\omega$ the expression (A.12) for $\Theta_1 - \psi$, we find, according to (2.22)

$$\begin{aligned} \Theta_2 - \psi &= d(\bar{Z}_1^\alpha f_i^\alpha dx^i) + *_4 d(\bar{Z}_1^\alpha f_i^\alpha dx^i) + d\bar{\beta}^\alpha |f_i^\alpha|^2 \\ &\quad - \frac{1}{2} \partial_v [\bar{\beta}^\alpha \wedge f_i^\alpha dx^i + *_4 d(\bar{\beta}^\alpha \wedge f_i^\alpha dx^i)], \end{aligned} \quad (\text{A.28})$$

which is of the same form as $\Theta_1 - \psi$ with the exchange of Z_1 with Z_2 ; it immediately follows that the 1-form a_2 exists and it is given by

$$a_2 = (\bar{Z}_1^\alpha - 1) f_i^\alpha dx^i - \bar{\gamma}_{1ik}^\alpha f_k^\alpha dx^i + \bar{\beta}^\alpha |f_i^\alpha|^2. \quad (\text{A.29})$$

The last equation to be verified is the one for \mathcal{F} , given in (2.23). From the ansatz (4.16)–(4.27) we see that \mathcal{F} is a linear combination of \bar{Z}_1^α , \bar{Z}_2^α and $\bar{\omega}^\alpha$, which are harmonic according to (4.12): hence $d *_4 d\mathcal{F} = 0$, and on the l.h.s. of equation (2.23) only the term containing $\dot{\omega}$ contributes. Thus the l.h.s. of (2.23) is

$$*_4 d *_4 \dot{\omega} = *_4 \partial_v [d *_4 ((\bar{Z}_1^\alpha + \bar{Z}_2^\alpha) f_i^\alpha dx^i) - \partial_v (d *_4 (\bar{\zeta}_{ki}^\alpha f_k^\alpha dx^i))], \quad (\text{A.30})$$

where a term proportional to $d *_4 \bar{\omega}^\alpha$ and one proportional to $d *_4 \bar{\beta}^\alpha$ have been dropped on account of (4.12). One has

$$*_4 d *_4 ((\bar{Z}_1^\alpha + \bar{Z}_2^\alpha) f_i^\alpha dx^i) = -\partial_i (\bar{Z}_1^\alpha + \bar{Z}_2^\alpha) f_i^\alpha = \partial_v (\bar{Z}_1^\alpha + \bar{Z}_2^\alpha), \quad (\text{A.31})$$

and

$$*_4 d *_4 (\bar{\zeta}_{ki}^{\alpha} \dot{f}_k^{\alpha} dx^i) = -\partial_k \bar{\zeta}_{ik}^{\alpha} \dot{f}_i^{\alpha} = -\bar{\beta}_i^{\alpha} \dot{f}_i^{\alpha}, \quad (\text{A.32})$$

where we have used that $\bar{\beta}^{\alpha} = - *_4 d \bar{\zeta}^{\alpha} = *_4 d *_4 \bar{\zeta}^{\alpha}$, as it follows from (4.12); hence

$$*_4 d *_4 \dot{\omega} = \partial_v^2 (\bar{Z}_1^{\alpha} + \bar{Z}_2^{\alpha} + \bar{\beta}_i^{\alpha} \dot{f}_i^{\alpha}). \quad (\text{A.33})$$

One the r.h.s. of (2.23) one finds

$$\partial_v^2 (Z_1 + Z_2) + \frac{1}{2} \partial_v^2 (h_{ii}) = \partial_v^2 (\bar{Z}_1^{\alpha} + \bar{Z}_2^{\alpha} + 2\bar{\beta}_i^{\alpha} \dot{f}_i^{\alpha} - \bar{\beta}_i^{\alpha} \dot{f}_i^{\alpha}) = *_4 d *_4 \dot{\omega}, \quad (\text{A.34})$$

which proves (2.23).

Appendix B. Coordinate shift

Let us consider the supergravity ansatz of Section 2 with a flat 4D base metric $h_{ij} = \delta_{ij}$. We perform the shift $x^i \rightarrow x^i - f^i(v)$ on the R^4 coordinates and rewrite the resulting 10D metric in the form dictated by the ansatz (2.2). As a result we obtain a new form for the 4D base metric and the other geometric data

$$ds_4^2 = (1 - \bar{\beta}'_k \dot{f}_k) dx^i dx^i + (\bar{\beta}'_j \dot{f}_i + \bar{\beta}'_i \dot{f}_j) dx^i dx^j + \frac{\bar{\beta}'_i \bar{\beta}'_j}{1 - \bar{\beta}'_k \dot{f}_k} |\dot{f}|^2 dx^i dx^j, \quad (\text{B.1})$$

$$\beta = \frac{\bar{\beta}'}{1 - \bar{\beta}'_k \dot{f}_k}, \quad (\text{B.2})$$

$$Z_I = \frac{\bar{Z}'_I}{1 - \bar{\beta}'_k \dot{f}_k}, \quad I = 1, 2, 4, \quad (\text{B.3})$$

$$\omega = \bar{\omega}' + \bar{\beta}' \left(\frac{\bar{\omega}'_i \dot{f}_i}{1 - \bar{\beta}'_k \dot{f}_k} + \frac{\bar{Z}'_1 \bar{Z}'_2}{\bar{\alpha}'(1 - \bar{\beta}'_k \dot{f}_k)^2} |\dot{f}|^2 \right) + \frac{\bar{Z}'_1 \bar{Z}'_2}{\bar{\alpha}'(1 - \bar{\beta}'_k \dot{f}_k)} \dot{f}_i dx^i, \quad (\text{B.4})$$

$$\mathcal{F} = \bar{\mathcal{F}}'(1 - \bar{\beta}'_k \dot{f}_k) - 2\bar{\omega}'_k \dot{f}_k - \frac{\bar{Z}'_1 \bar{Z}'_2}{\bar{\alpha}'(1 - \bar{\beta}'_k \dot{f}_k)} |\dot{f}|^2, \quad (\text{B.5})$$

where the quantities on the l.h.s. define the solution after the shift, while the barred and primed quantities on the r.h.s. are the original geometric data (i.e. those in the frame where the base metric is flat) evaluated at the point $x - f^i(v)$. By repeating the same change of variables on the other supergravity fields we obtain

$$a_1 = (1 - \bar{\beta}'_k \dot{f}_k) a'_1 + \bar{\beta}' \bar{a}'_{1k} \dot{f}_k + \bar{Z}_2 \left(\dot{f}_i dx^i + \frac{\bar{\beta}'}{1 - \bar{\beta}'_k \dot{f}_k} |\dot{f}|^2 \right) - \bar{\gamma}'_{2ij} dx^i \dot{f}^j, \quad (\text{B.6})$$

$$a_4 = (1 - \bar{\beta}'_k \dot{f}_k) \bar{a}_4 + \bar{\beta}' \bar{a}'_{4k} \dot{f}_k + \bar{Z}_4 \left(\dot{f}_i dx^i + \frac{\bar{\beta}'}{1 - \bar{\beta}'_k \dot{f}_k} |\dot{f}|^2 \right) - \bar{\delta}'_{2ij} dx^i \dot{f}^j, \quad (\text{B.7})$$

$$\gamma_2 = \bar{\gamma}'_2 + \bar{\gamma}'_{2ij} \dot{f}_i \frac{\bar{\beta}'}{1 - \bar{\beta}'_k \dot{f}_k} \wedge dx^j, \quad (\text{B.8})$$

$$\delta_2 = \bar{\delta}'_2 + \bar{\delta}'_{2ij} \dot{f}_i \frac{\bar{\beta}'}{1 - \bar{\beta}'_k \dot{f}_k} \wedge dx^j, \quad (\text{B.9})$$

$$\begin{aligned}
x_3 = & \bar{x}'_3(1 - \bar{\beta}'_k \dot{f}_k) + \frac{1}{2} \bar{\beta}' \wedge \bar{x}'_{3ijk} \dot{f}_k dx^i \wedge dx^j + \bar{Z}'_4 \bar{\gamma}'_2 \wedge \left(\dot{f}_i dx^i + \frac{\bar{\beta}'}{1 - \bar{\beta}'_k \dot{f}_k} |\dot{f}|^2 \right) \\
& + \bar{Z}'_4 \bar{\gamma}'_{2ij} \dot{f}_i dx^j \wedge \frac{\bar{\beta}'}{1 - \bar{\beta}'_k \dot{f}_k} \wedge \dot{f}_k dx^k.
\end{aligned} \tag{B.10}$$

It is straightforward to check that Eqs. (5.17)–(5.28) are reproduced by choosing the quantities on the r.h.s. as done in (5.1)–(5.12) and then by linearizing the result in β^{D1D5} and ω^{D1D5} . As discussed in Section 5, this keeps the dependence on the D1 and D5 charges Q_1 and Q_5 exact and includes only the first-order backreaction due to the KK-monopole dipole charge β^{D1D5} , to ω^{D1D5} and to the other objects that are related to β^{D1D5} or ω^{D1D5} by the supergravity equations.

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